SOE Time Complexity:

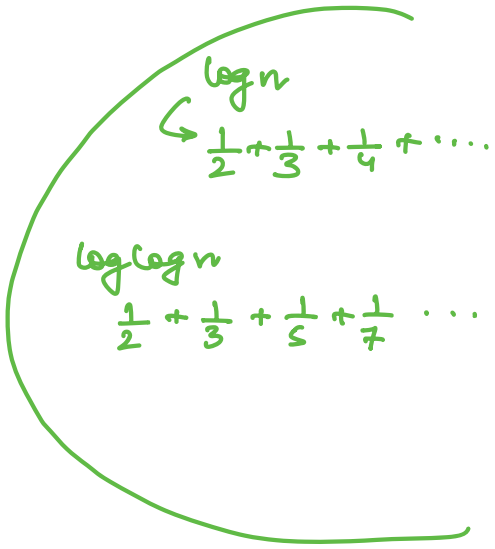
table: 2 table: 3 ..... $\sqrt{n}$

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7}$$

$$n\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \cdots\right)$$

$\log \log n$

$O(n \log \log n)$



$n = 10^{10^{10}}$

$\log n = 10^{10}$

$\log \log n = 10$

Recursive Program Time Complexity

- Recurrence Relation
- Shortcut
- Master Theorem

```
TOH(n, S, D, H)
{
  TOH(n-1, S, H, D)
  pf(move n from s to D)
  TOH(n-1, H, D, S)
}
```

Recurrence Relation:

$$T(n) = T(n-1) + 1 + T(n-1)$$

$$T(n) = 2T(n-1) + 1$$

$$2T(n-1) = 2^2 T(n-2) + 1 \cdot 2$$

$$2^2 T(n-2) = 2^3 T(n-3) + 1 \cdot 2^2$$

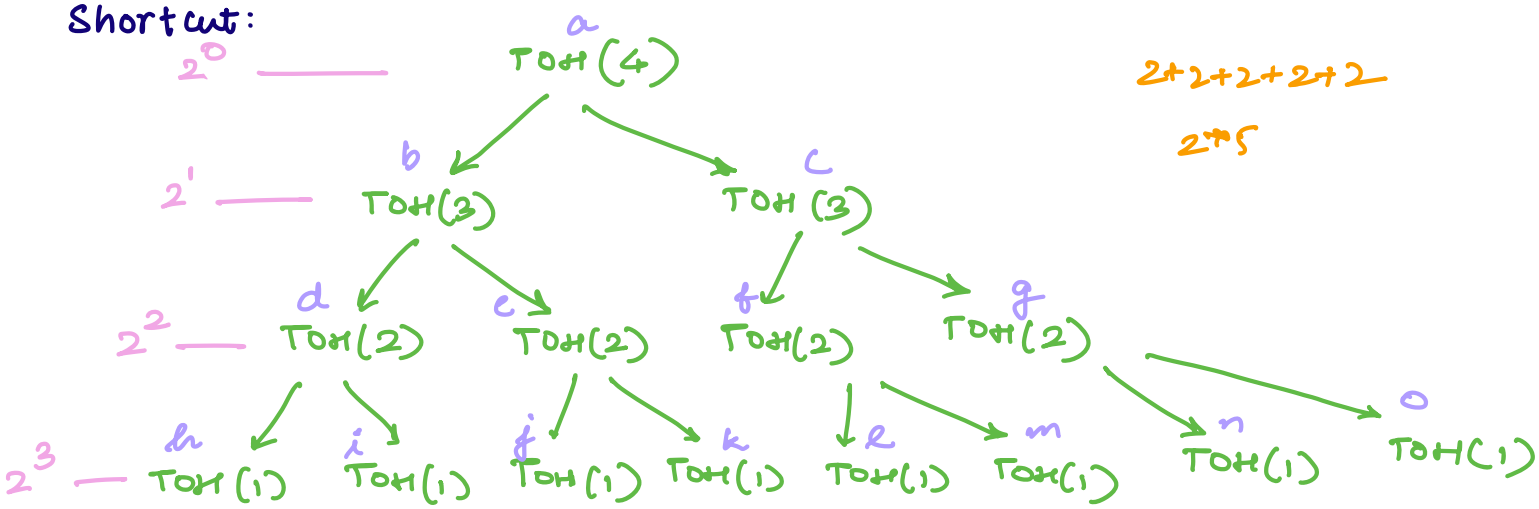
⋮

$2^{n-1}\ T(n-(n-1)) = 1 \cdot 2^{n-1}$

$T(n) = 2^0 + 2^1 + 2^2 + \ldots\ldots 2^{n-1}$ $\qquad a\left(\frac{r^n-1}{r-1}\right)$

$= 1\left(\frac{2^n-1}{2-1}\right) = \underbrace{2^n-1}_{f(n)} \leq \underbrace{1 \cdot 2^n}_{c\ g(n)} = O(2^n)$

Shortcut:



Time: $a+b+c+d+e+f+g+h+i+j+k+l+m+n+o$

If in every frame same amount of work then,

no. of fx^n frame * work

$2^0 + 2^1 + 2^2 + \ldots\ldots 2^{n-1}$

$2^n - 1$

$TC = (2^n-1) * 1 = 2^n - 1 = O(2^n)$ (1 → print)

Shortcut: if ≥2 Rec calls & same work in each fx^n frame

TC: no. of fx^n frames * work

(no. of calls * work in each)

masters theorem

$$T(n) = a\,T\left(\frac{n}{b}\right) + n^k \log^p n$$

$a \geq 1$, $b > 1$, $k \geq 0$, p real no.

1) if $a > b^k$ then $T(n) = \Theta\left(n^{\log_b a}\right)$

2) if $a = b^k$

   a) if $p > -1$ then $T(n) = \Theta\left(n^{\log_b a} \log^{p+1} n\right)$

   b) if $p = -1$ then $T(n) = \Theta\left(n^{\log_b a} \log\log n\right)$

   c) if $p < -1$ then $T(n) = \Theta\left(n^{\log_b a}\right)$

3) if $a < b^k$

   a) if $p \geq 0$ then $T(n) = \Theta\left(n^k \log^p n\right)$

   b) if $p < 0$ then $T(n) = O(n^k)$

$T(n) = 2\,T\left(\frac{n}{4}\right) + n^2$

$a = 2$
$b = 4$
$k = 2$
$p = 0$

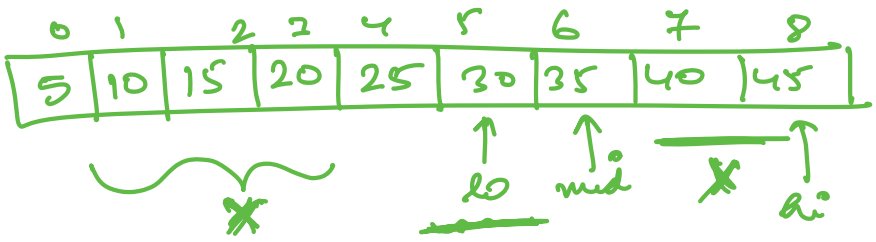
$a < b^k$
$2 < 4^2$

$n^k \log^p n = n^2 (\log n)^0 = n^2$

Binary Search:

sorted

item: 30 ?



lo=0
hi=8
mid = 0+8/2 = 4
mid 25 < item 30
sorted
lo = mid+1
= 5

lo=5
hi=8
mid = 6
mid 35 > item 30
hi = mid-1
= 5

lo=5
hi=5
mid 5
mid item
30 == 30
5 index found

mid = lo + (hi-lo)/2

lo = INT_MAX - 50
hi = INT_MAX - 20

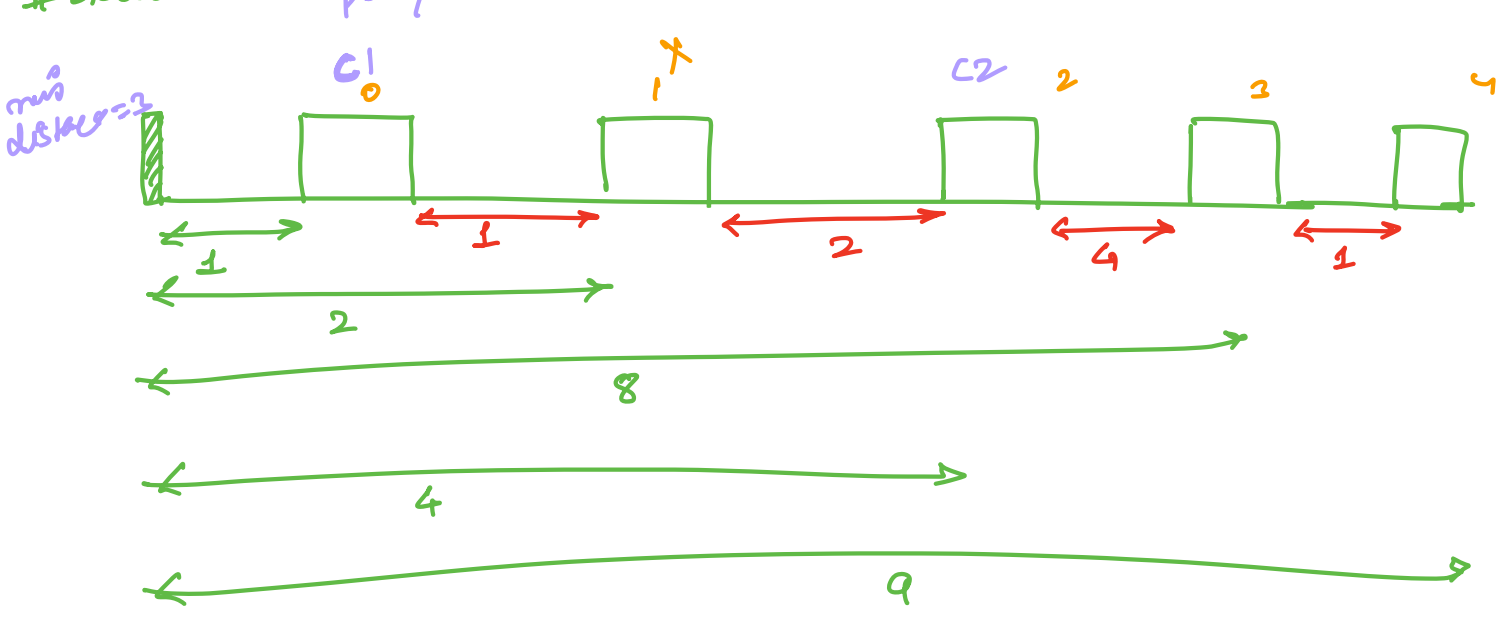
4
4
5236

https://www.spoj.com/problems/AGGRCOW/

Aggressive Cows

maximize the min distance b/w cows.

cows = 3
stalls = 5



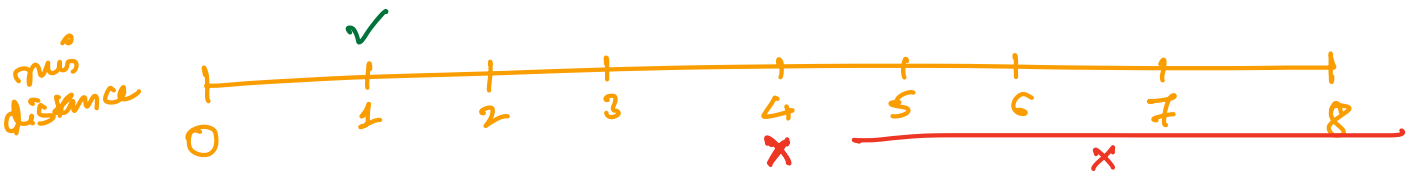
min distance = 1	C1	C2	C3

min distance = 2 — not — C1 C2 C3

min distance = 3 — C1 C2 C3

min distance = 4 — ✗ — C1 C2

not able to place all cows



lo = 0
hi = 8
mid = 4
min distance 4 placed cows?
No
LHS
hi = mid - 1
= 3

lo = 0
hi = 3
mid = 1
min distance 1 placed cows?
Yes
RHS
lo = mid + 1
= 2

lo = 2
hi = 3
mid = 2
Is it possible?
Yes
RHS
lo = mid + 1
= 3

lo = 3
hi = 3
mid = 3
Is it possible?
Yes
RHS
lo = 4

lo = 4
hi = 3